3.1. Introduction

Any separation process involves three types of transport phenomena: heat transfer, fluid flow and mass transfer. Most bioseparation processes being isothermal or nearly isothermal in nature, the role of heat transfer is not as significant as in conventional chemical separations. The role of fluid flow is much more significant in comparison and will be dealt with in this text where relevant on a need to know basis. The role of mass transfer is perhaps most significant in bioseparations and hence an entire chapter is devoted to it.

Mass transfer as the term implies deals with transport of material. However, it is distinctly different from fluid flow which also deals with transport of material. A simple example of mass transfer is the movement of a scent from one end of a room to the other. Mass transfer basically deals with transport of species within a medium or across an interface, i.e. from one medium to another. The medium could be stationary or mobile. There are two types of mass transfer:

1. Purely diffusive mass transfer (or molecular diffusion)
2. Convective mass transfer

Molecular diffusion is governed by a random walk process and involves the transport of molecules from a region of high concentration to one where its concentration is lower. Steady state molecular diffusion of species \( A \) (e.g. sucrose) in a diffusion medium \( B \) (e.g. water) can be expressed by Fick’s first law (see Fig. 3.1):

\[
J_A = -D_{AB} \frac{dc_A}{dx} \tag{3.1}
\]

Where

\( J_A \) = flux of \( A \) in \( B \) (kg-moles/m\(^2\).s)
\[ D_{AB} = \text{diffusivity of } A \text{ in } B \text{ (m}^2\text{/s)} \]

\[ c_A = \text{concentration of } A \text{ (kg-moles/m}^3\text{)} \]

\[ x = \text{length along direction of diffusion (m)} \]

Higher concentration of \( A \)

Lower concentration of \( A \)

![Fig. 3.1 Molecular diffusion](image)

Convective mass transfer takes place in flowing fluids, particularly when the flow is turbulent in nature, i.e. where there are eddies. An equation representing steady state convective mass transfer of species \( A \) (e.g. a protein) in a transfer zone within a flowing fluid (e.g. water) is shown below (see Fig. 3.2):

\[ N_A = k_A \Delta c_A \]

Where

\[ N_A = \text{flux of } A \text{ (kg-moles/m}^2\text{.s)} \]

\[ k_A = \text{mass transfer coefficient of } A \text{ within the transfer zone (m/s)} \]

\[ \Delta c_A = \text{concentration difference across transfer zone (kg-moles/m}^3\text{)} \]

In equation (3.2) the flux is denoted by \( N_A \) to distinguish it from \( J_A \) which is generally used for flux purely due to molecular diffusion. In convective mass transfer, flux of solute takes place due to a combination of molecular diffusion and eddy diffusion.

### 3.2. Molecular diffusion in liquid medium

Molecular diffusion in the context of bioseparations mainly involves the transport of dissolved species in a liquid medium. An example of this is the transport of an antibiotic from an aqueous solution to the surface of
an ion-exchange resin. Gaseous diffusion is far less important in bioseparation except in very specific types of separation e.g. freeze drying, pervaporation and molecular distillation. Molecular diffusion in liquid medium is significantly slower than that in a gaseous medium and hence the diffusion coefficients are significantly lower.

If we consider a simple case of molecular diffusion e.g. that of an amino acid \((A)\) in water \((B)\), the flux of the \(A\) from point 1 to 2 in the liquid medium is given by (see Fig. 3.3):

\[
J_A = -D_{AB} \frac{dc_A}{dx} = -D_{AB} \frac{c_{A2} - c_{A1}}{x_2 - x_1} \tag{3.3}
\]

Similarly the flux of \(B\) from 2 to 1 is given by:

\[
J_B = D_{BA} \frac{dc_B}{dx} = D_{BA} \frac{c_{B1} - c_{B2}}{x_1 - x_2} \tag{3.4}
\]

The diffusion coefficient of \(A\) in \(B\) i.e. \(D_{AB}\) is the same as the diffusion coefficient of \(B\) in \(A\), i.e. \(D_{BA}\). In a two-component system where the total molar concentration at any point is largely constant:

\[
- \frac{dc_A}{dx} = \frac{dc_B}{dx} \tag{3.5}
\]

Hence:

\[
J_A = -J_B \tag{3.6}
\]

This is referred to as equimolar counter-diffusion. This means that the molar flux of the amino in a certain direction is matched by the molar flux of water in the opposite direction.
3.3. Measurement of diffusivity

The diffusivity of a solute in a liquid medium can be experimentally measured using various techniques. A commonly used technique is based on the diffusion cell (see Fig. 3.4) which consists of two well mixed chambers having the same volume which are separated by a porous membrane. Initially the two chambers are filled with the liquid medium. It is ensured that the pores of the membrane are also completely filled with the same liquid. At time $t = 0$, the liquid in one of the chambers (say chamber 1), is replaced with a solution of known concentration of the solute. The concentration of the solute in one or both chambers of the diffusion cell is/are then monitored, and based on the change in solute concentration with time its diffusivity can be determined using the following equation:

$$D_{AB} = \frac{V \delta \tau}{2 \varepsilon \alpha t} \ln \left( \frac{c_1^0 - c_2^0}{c_1 - c_2} \right)$$  \hspace{1cm} (3.7)

Where

- $V$ = volume of a chamber ($m^3$)
- $\delta$ = thickness of the membrane (m)
\tau = \text{tortuosity of the membrane (-)}
\varepsilon = \text{porosity of the membrane (-)}
\sigma = \text{area of the membrane (m}^2\text{)}
\eta = \text{time (s)}
\text{c}_1 = \text{solute concentration in chamber 1 (kg-mole/m}^3\text{)}
\text{c}_2 = \text{solute concentration in chamber 2 (kg-mole/m}^3\text{)}

The superscript 0 represents initial value i.e. at \( t = 0 \).

There are several other techniques for measuring diffusivity, some of these being variants of the technique discussed above.

\[ D_{AB} = \frac{9.96 \times 10^{-16} T}{\mu V_A^{1/3}} \]  

(3.8)

Where

\( T \) = absolute temperature (K)
\( \mu \) = viscosity of the liquid medium (kg/m s)

3.4. Estimation of diffusivity

The diffusivity of a solute in a liquid medium at a particular temperature can also be estimated using different mathematical correlations. These correlations link diffusivity to solute and liquid properties such as molar volume, molecular weight and liquid viscosity. The three most widely used correlations are:

Stokes-Einstein correlation:
\[ V_A = \text{solute molar volume at its normal boiling point (m}^3/\text{kg-mole)} \]

Wilke-Chang correlation:

\[ D_{AB} = \frac{1.173 \times 10^{-16} (\phi M_B)^{1/2} T}{\mu V_A^{0.6}} \]

\( (3.9) \)

Where

\( \phi \) = association parameter (-) and has a value of 2.6 for water

\( M_B \) = molecular weight of the liquid medium (kg/kg-mole)

Polson correlation:

\[ D_{AB} = \frac{9.40 \times 10^{-15} T}{\mu M_A^{1/3}} \]

\( (3.10) \)

Where

\( M_A \) = molecular weight of the solute (kg/kg-mole)

Diffusivity of electrolytes can be estimated using the Nernst-Haskell correlation:

\[ D_{AB} \approx \frac{8.928 \times 10^{-10} T (1/n_+ + 1/n_-)}{(1/\lambda_+ + 1/\lambda_-)} \]

\( (3.11) \)

Where

\( n_+ \) = valency of the cation

\( n_- \) = valency of the anion

\( \lambda_+ \) = ionic conductance of the cation

\( \lambda_- \) = ionic conductance of the anion

The correlation shown above gives diffusivity in cm\(^2\)/s. \( \lambda \) and \( n \) values for different cations and anions can be obtained from standard tables of physical properties.

The diffusion concepts discussed so far are based on simple systems, i.e. the solution of a single solute. Most systems handled in bioseparation processes are complex and hence the correlations discussed above have to be appropriately modified to account for specific system related effects.

**Example 3.1**

Estimate the diffusivity of the protein lysozyme in water at 25 degrees centigrade.

**Solution**

The diffusivity of a solute can be calculated from its molecular weight using Polson correlation i.e. equation (3.10). From Table 2.2 the
molecular weight of lysozyme is 14,100 kg/kg-mole. The viscosity of water at 25 degrees centigrade is 0.001 kg/m s. Therefore:

\[ D_{\text{lysozyme}} = \frac{9.40 \times 10^{-15} \times 298}{0.001 \times 14100^{1/3}} \text{ m}^2/\text{s} = 1.16 \times 10^{-10} \text{ m}^2/\text{s} \]

3.5. Diffusion of solutes in dense solid

Solute molecules can diffuse through dense solid medium after dissolving in it. An example of this is the diffusion of ions through dense membranes. The molecules of the solid medium do not counter-diffuse on account of their limited mobility. However, Fick’s law can still be used to describe the diffusion of solute molecules in a solid medium.

3.6. Diffusion of solutes in porous solid

Solute molecules can diffuse through the pores present in porous solids. In order for this to happen, the pores have to be filled with some liquid medium. Therefore no diffusion takes place through the solid material itself. All it does is hold the liquid medium in place. However, the solid material can have an influence on the diffusion within the liquid medium. It can for instance increase the effective diffusion path length of the solute if the pores are tortuous in nature. When the pores have dimension of the same order of magnitude as the solute, the pore wall can cause hindrance to diffusion. An example of un-hindered diffusion in a porous medium is the transport of sodium chloride through a microfiltration membrane (which has micron sized pores) while an example of hindered diffusion is the transport of albumin through an ultrafiltration membrane (which has nanometer sized pores). The steady state equation for unhindered diffusion of a solute from point 1 and 2 within a slab of porous solid is given by:

\[ J_A = \frac{\varepsilon D (c_{A1} - c_{A2})}{\tau(x_2 - x_1)} \]  

(3.12)

Where

- \( D \) = diffusivity of the solute in the liquid within the pores (m\(^2\)/s)
- \( \varepsilon \) = porosity of the medium (-)
- \( \tau \) = tortuosity of the medium (-)

The hindered diffusion of a solute through a porous solid from point 1 to 2 is given by:
\[ J_A = \frac{D_{\text{eff}} \varepsilon (c_{A1} - c_{A2})}{\tau (x_2 - x_1)} \]  \tag{3.13}

Where

\[ D_{\text{eff}} \] = effective hindered diffusivity (m\(^2\)/s)

The effective hindered diffusivity of a solute in a pore can be obtained by:

\[ D_{\text{eff}} = D \left( 1 - \frac{d_s}{d_p} \right)^4 \]  \tag{3.14}

Where

\[ d_s \] = solute diameter (m)
\[ d_p \] = pore diameter (m)

**Example 3.2**

A membrane having a porosity of 0.75, an average pore size of 1 micron, a surface area of 2 cm\(^2\) and a thickness of 0.1 mm separates two water-filled, well-mixed chambers each having volume of 10 ml. The content of one of the chambers was replaced with 10 ml of 1 mg/ml human albumin solution at \( t = 0 \). Calculate the solute concentration in the other chamber after 50 minutes. Assume that the pores of the membrane are all aligned normal to the membrane surface and that there is no convective flow of solvent through the membrane.

**Solution**

In this problem we assume that the diffusion of albumin through the pores is not hindered. This can be confirmed by using equation (3.14). The diameter of albumin is 7.2\( \times \)10\(^{-3}\) microns (ref. Table 2.1). Therefore:

\[ D_{\text{eff}} = D \]

The diffusivity of albumin is 5.94\( \times \)10\(^{-11}\) m\(^2\)/s (ref. Table 2.3). Using equation (3.7), we can write:

\[ 5.94 \times 10^{-11} = \frac{10 \times 10^{-6} \times 0.1 \times 10^{-3}}{2 \times 0.75 \times 2 \times 10^{-4} \times 3000} \ln \left( \frac{1}{C_1 - C_2} \right) \]  \tag{3.a}

In this equation we have replaced the molar concentration with the mass concentration since the molecular weight cancels out between the numerator and denominator of the term within parenthesis. The total
amount of solute (albumin) in the system remains constant. Therefore we can write:

\[ C^0_i V = C_i V + C_2 V \]

Therefore:

\[ 1 \times 10^{-6} = (C_1 + C_2) \times 10^{-6} \]  \hspace{1cm} (3.b)

Solving equations (3.a) and (3.b) simultaneously, we get:

\[ C_1 = 0.973 \text{ mg/ml} \]
\[ C_2 = 0.027 \text{ mg/ml} \]

**Example 3.3**

Glucose is diffusing at 25 degrees centigrade in water within a porous medium having a porosity of 0.5, tortuosity of 1.8 and average pore diameter of \(8.6 \times 10^{-3}\) microns. Determine the steady state flux of glucose between two points within the medium separated by a distance of 1 mm and having concentrations 1.5 g/l and 1.51 g/l respectively.

**Solution**

Table 2.2, gives the molecular weight of glucose as being 180 kg/kg-mole. The diffusivity of glucose at 25 degrees centigrade can be determined using Poisson correlation i.e. equation (3.10):

\[ D = \frac{9.40 \times 10^{-15} \times 298}{0.001 \times (180)^{1/3}} \text{ m}^2/\text{s} = 4.96 \times 10^{-10} \text{ m}^2/\text{s} \]

The size of glucose can be obtained from Table 2.1. The effective diffusivity of glucose in the porous structure can be obtained using equation (3.14):

\[ D_{eff} = 4.96 \times 10^{-10} \left(1 - \frac{8.6 \times 10^{-4}}{8.6 \times 10^{-3}}\right)^4 \text{ m}^2/\text{s} = 3.25 \times 10^{-10} \text{ m}^2/\text{s} \]

The steady state flux of glucose can be obtained using equation (3.13):

\[ J_d = \frac{3.25 \times 10^{-10} \times 0.5 \times 0.01}{1.8 \times 0.001 \times 180} \text{ kg-moles/m}^2\text{s} = 5.02 \times 10^{-12} \text{ kg-moles/m}^2\text{s} \]

**3.7. Convective mass transfer**

Convective mass transfer is observed in flowing fluids e.g. transport of a solute in a liquid flowing past a solid surface (see Fig. 3.5), or transport of a solute in a liquid flowing past another immiscible liquid (see Fig. 3.6). An example of the first type is the transfer of urea from blood
towards the surface of a dialyser membrane in haemodialysis. An example of the second type is the transfer of penicillin G within filtered aqueous media flowing past an organic solvent in a liquid-liquid extractor. When a liquid flows past a solid surface a stagnant boundary liquid layer is formed close to the surface. Similarly, when two liquids flow past one another, two boundary liquid layers are generated on either sides of the interface. Within these boundary layers, the transport of solute mainly takes place by molecular diffusion. If the flow of liquid is laminar, the transfer of solute in the directions indicated in Figs. 3.5 and 3.6 would be by molecular diffusion. However, if the flow were turbulent in nature, mass transfer would take place by a combination of molecular diffusion and eddy diffusion. This is referred to as convective mass transfer. The flux equation for convective mass transfer is:

$$ N_A = -(D + E) \frac{dc_A}{dx} $$

(3.15)

Where

$E = \text{eddy diffusivity (m}^2/\text{s})$

Equation (3.15) can be written as:

$$ N_A = k_A \Delta c_A $$

(3.16)

Where

$k_A = \text{mass transfer coefficient (m/s)}$

**Example 3.4**

An aqueous solution of human immunoglobulin G (at 4 degrees centigrade) is being pumped through a tube having a diameter of 1 mm. The mass transfer coefficient for the protein in the radial direction was found to be $1 \times 10^{-6}$ m/s. Comment on this value vis-à-vis the diffusivity of the protein. Estimate the eddy diffusivity of the flowing system.

**Solution**

From Table 2.2, the molecular weight of immunoglobulin G is 155,000 kg/kg-mole. Its diffusivity in water can be calculated using Polson correlation:

$$ D_{AB} = \frac{9.40 \times 10^{-15} \times 277}{0.001 \times (155,000)^{1/3}} \text{ m}^2/\text{s} = 4.85 \times 10^{-11} \text{ m}^2/\text{s} $$

In this problem the diffusion length is the radius of the tube which is 0.5 mm. If solute transport is due to diffusion alone, the mass transfer
coefficient is obtained by dividing the diffusivity by the diffusion length. Therefore:

\[ k_D = \frac{4.85 \times 10^{-11}}{0.5 \times 10^{-3}} \text{ m/s} = 9.69 \times 10^{-8} \text{ m/s} \]

This is significantly lower than the observed mass transfer coefficient. Therefore there is some eddy diffusivity involved in solute transport. Eddy diffusivity can be obtained from equations (3.15) and (3.16):

\[ E = (1 \times 10^{-6} \times 0.5 \times 10^{-3}) - 4.85 \times 10^{-11} \text{ m}^2/\text{s} = 4.52 \times 10^{-10} \text{ m}^2/\text{s} \]
3.8. Experimental determination of mass transfer coefficient

Mass transfer coefficient of a solute in a flowing liquid can be determined by carrying out steady-state experiments based on equation (3.16). The general approach in these experiments is to measure the amount of solute transferred in a given period of time across a given surface area for a particular concentration difference across the transfer zone. Solute mass transfer coefficient in a liquid flowing within a tube can be determined by coating the solute over the inner wall of the tube followed by measurement of the amount of solute removed. The average flux across the mass transfer zone can be calculated from:

\[ N_A = \frac{m}{a \Delta t} \]  
\[ k_A = \frac{m}{a \Delta t \Delta c_A} \]

Example 3.5
Water is flowing past one of the rectangular sides of a slab of benzoic acid. The surface area exposed to water is 0.01 m² and it is estimated that in 600 seconds, 5 \times 10^{-4} kg of benzoic acid is lost from the slab by dissolution. If the molecular weight of benzoic acid is 121.1 kg/kg-mol, calculate its average molar flux. If the solubility of benzoic acid in water is 0.2 kg/m³, what is its mass transfer coefficient? Assume that the concentration of benzoic acid in the bulk flowing water is negligible.

Solution
The amount of solute transferred in 600 s is:

\[ m = \frac{5 \times 10^{-4}}{121.1} \text{ kg-moles} = 4.13 \times 10^{-6} \text{ kg-moles} \]

The flux can be calculated using equation (3.17):

\[ N_A = \frac{4.13 \times 10^{-6}}{0.01 \times 600} \text{ kg-moles/m}^2 \text{ s} = 6.9 \times 10^{-7} \text{ kg-moles/m}^2 \text{ s} \]
Assuming that the benzoic acid concentration on the surface of the slab is the same as its solubility, the concentration difference across the transfer zone is:

\[
\Delta c = \frac{0.2}{121.1} \text{kg-moles/m}^3 = 1.65 \times 10^{-3} \text{ kg-moles/m}^3
\]

The mass transfer coefficient can be calculated using equation (3.18):

\[
k_\text{A} = \frac{4.13 \times 10^{-6}}{0.01 \times 600 \times 1.65 \times 10^{-3}} \text{ m/s} = 4.17 \times 10^{-4} \text{ m/s}
\]

### 3.9. Estimation of mass transfer coefficient

The mass transfer coefficient can be estimated using numerical correlations which are based on heat-mass transfer analogy. These correlations typically have three or more dimensionless groups. The three dimensionless groups that are always present in these correlations are the Reynolds number \(N_{Re}\), the Sherwood number \(N_{Sh}\) and the Schmidt number \(N_{Sc}\):

\[
N_{Re} = \frac{du \rho}{\mu}
\]  
(3.19)

Where

- \(d\) = hydraulic diameter of the flow passage (m), e.g. tube diameter
- \(u\) = velocity of flowing liquid (m/s)
- \(\rho\) = density (kg/m\(^3\))
- \(\mu\) = solution viscosity (kg/m s)

\[
N_{Sh} = \frac{k_A d}{D}
\]  
(3.20)

Where

- \(D\) = solute diffusivity (m\(^2\)/s)

\[
N_{Sc} = \frac{\mu}{\rho D}
\]  
(3.21)

The general form of such dimensionless correlations is:

\[
N_{Sh} = a N_{Re}^b N_{Sc}^c
\]  
(3.22)

Where \(a\), \(b\) and \(c\) are constants

Some dimensionless correlations of this type will be discussed in the chapter on membrane based bioseparation.
3.10. Inter-phase mass transfer

So far we have discussed mass transfer within a medium. The transport of a solute from one medium to another across an interface is called inter-phase or interfacial mass transfer. Such type of material transport is quite common in separation processes such as liquid-liquid extraction, leaching, chromatography and membrane separation. As an example of inter-phase mass transfer, the transport of a solute from a liquid to another immiscible liquid will be discussed here. For the sake of simplicity the two liquids are assumed to be stagnant thus eliminating the need for considering boundary layers.

Fig. 3.7 shows the steady-state solute concentration profile close to the interface between the two liquids. The solute flux across the interface which takes place from liquid 1 to liquid 2 is given by:

\[ J = D_1 \frac{(c_1 - c_{i1})}{x_1} = D_2 \frac{(c_{i2} - c_2)}{x_2} \]  

(3.23)

Where

- \( c_1 \) = solute concentration at \( x_1 \) distance from interface in liquid 1
- \( c_2 \) = solute concentration at \( x_2 \) distance from interface in liquid 2
- \( c_{i1} \) = interfacial solute concentration in liquid 1
- \( c_{i2} \) = interfacial solute concentration in liquid 2
- \( D_1 \) = diffusivity in liquid 1
- \( D_2 \) = diffusivity in liquid 2

The interfacial solute concentrations in the two liquids are linked by the partition coefficient:

\[ c_{i2} = K c_{i1} \]  

(3.24)

Where

- \( K \) = partition coefficient (-)

3.11. Unsteady state mass transfer

The discussion so far has been based on steady-state mass transfer, i.e. where the concentrations at various locations within the transfer zone do not change with time. However, in many separations, particularly in rate processes, these concentrations do change with time and hence it is important to understand what happens in an unsteady state mass transfer process.
The general equation for unsteady state molecular diffusion in 3-dimension for the Cartesian co-ordinate system is of the form shown below:

\[
D \left( \frac{\partial^2 c_A}{\partial x^2} + \frac{\partial^2 c_A}{\partial y^2} + \frac{\partial^2 c_A}{\partial z^2} \right) = \frac{\partial c_A}{\partial t}
\]  

(3.25)

Depending on the geometry of the system under consideration, an appropriate partial differential equation which could be a modified form of equation (3.25) needs to be set up. This is then solved taking into consideration appropriate initial and boundary conditions. As a case study, the unsteady state diffusion of a solute in a semi-infinite gel is discussed here. If a solution having a solute concentration $c_A$ is suddenly brought into contact with a thick slab of gel within which the initial concentration of the solute is $c_{A0}$, the concentrations within the slab at various locations will change with time as shown in Fig. 3.8. In order to simplify the problem we assume that the concentration of the solute in the solution adjacent to the slab remains constant. We also assume that the solute concentration is identical on either sides of the solution-slab interface, i.e. its partition coefficient is unity. This is an example of mass transfer in 1-dimension, i.e. along the x-axis. Equation (3.25) reduces to:
Initial condition is:
At $t = 0$, for all $x$, $c_A = c_{A0}$

Boundary conditions are:
At $x = 0$, for $t > 0$, $c_A = c_{A1}$ i.e. $c_A(0,t) = c_{A1}$
At $x = \infty$, for all $t$, $c_A = c_{A0}$, i.e. $c_A(\infty,t) = c_{A0}$

Solving equation (3.26) we get:

$$\frac{c_{A1} - c_A}{c_{A1} - c_{A0}} = \text{erf} \left( \frac{x}{2\sqrt{D_{Ab}t}} \right)$$

(3.27)

Using this equation the solute concentration at any point $x$ at any time $t$ can be calculated. The flux of solute across the interface at any time $t$ is given by:

$$J_A(0,t) = \frac{D_{Ab}}{\pi t} \left( c_{A1} - c_{A0} \right)$$

(3.28)

**Example 3.6**

A thick gel slab of agar is suddenly exposed on one side to a well-mixed aqueous solution of an antibiotic, the purpose of this being to imbibe the antibiotic within the gel. The concentration of the antibiotic solution is 0.001 kg-moles/m$^3$. If we assume that the concentration in the well-mixed solution does not change appreciably in the time during which this experiment is carried out determine the flux of the antibiotic across the
gel-water interface after 5 minutes, given that its diffusivity in the gel is $8 \times 10^{-11}$ m$^2$/s.

**Solution**

The flux of the antibiotic across the interface can be calculated using equation (3.28):

$$J_A(0,600) = \sqrt{\frac{8 \times 10^{-11}}{3.142 \times 300}} (0.001 - 0) \text{ kg-moles/m}^2\text{s}$$

$$= 2.91 \times 10^{-11} \text{ kg-moles/m}^2\text{s}$$

### 3.12. Equilibrium and rate processes

Transfer of species from one zone of a continuum to another or indeed across an interface continues to take place until some form of equilibrium is established. This equilibrium could be defined in terms of chemical potential, concentration or other appropriate parameters. Some separation processes are carried out until equilibrium has been achieved e.g. liquid-liquid extraction of penicillin G from fermentation media to methyl isobutyl ketone. Such processes are referred to as equilibrium processes (Fig. 3.9).

![Fig. 3.9 Equilibrium process: Extraction of penicillin G](image)

However in many separations, the nature of the process is such that no identifiable equilibrium is reached. An example of this is haemodialysis where maintenance of a concentration gradient throughout the process is essential. Such processes are referred to as non-equilibrium or rate processes (Fig. 3.10).
**Exercise problems**

3.1. An aqueous solution of lysozyme (MW 14,100 kg/kg-mole) at a temperature of 25 degrees centigrade is being pumped at a flow rate of 5000 ml/min through a short tube having a diameter of 10 mm. The concentration of lysozyme in the feed solution is 1 g/l while its concentration adjacent to the tube wall is practically zero on account of the rapid adsorption of the protein on the wall of the tube. The steady state flux of lysozyme in the radial direction within the tube was found to be $1.3 \times 10^{-11}$ kg-mole/m².s. If molecular diffusion alone would have taken place, what would have been the steady state flux in the radial direction?

3.2. A membrane having a porosity of 0.3, average pore size of 0.01 micron, average pore tortuosity of 1.1, surface area of 2 cm² and thickness of 0.1 mm separates two water-filled, well-mixed chambers one having a volume of 10 ml and the other having a volume of 20 ml. The content of the 10 ml chamber was replaced with 10 ml of 1 mg/ml human albumin solution at $t = 0$. Calculate the albumin concentration in the other chamber after 300 minutes. Assume that there is no convective flow of solvent through the membrane. In this problem you cannot assume that the concentration in the 10 ml chamber does not change significantly with time. This experiment was carried out at 20 degrees centigrade.

3.3. A thick slab of agar containing 0.05 g/l of lysine is prepared in a Petri dish. The portion on top of the gel was suddenly flooded with a 1 g/l lysine solution. If we assume that the amino acid concentration in the solution does not change significantly with time determine the flux of lysine across the gel-water interface after
600 seconds. Predict the concentration of lysine at a distance of 5 mm from the interface at that time, given that its diffusivity in the gel is $6.5 \times 10^{-9} \text{ m}^2/\text{s}$.

3.4. Human immunoglobulin G (HIgG) is diffusing through a porous medium having a porosity of 0.5, tortuosity of 1.8 and average pore diameter of 0.45 microns. Protein samples were collected from two points within the medium separated by a distance of 20 mm and the ultraviolet light absorbance of these samples as measured with a spectrophotometer having a sample path length of 0.2 cm were found to be 0.010 and 0.012 respectively. If the specific absorbance of HIgG is known to be 15590 m$^3$/kg-moles.cm, calculate the steady state flux of this protein between the two points.

References


